Basic Principles of Monte Carlo Simulation

- A widely used and easy methodology to calculate integrals.
- Monte Carlo Simulations have been developed in the 1930s.
- It is based on the idea of trial and error.
- Monte Carlo can be called exact method since the result will converge for $n \rightarrow \infty$ to the exact result.

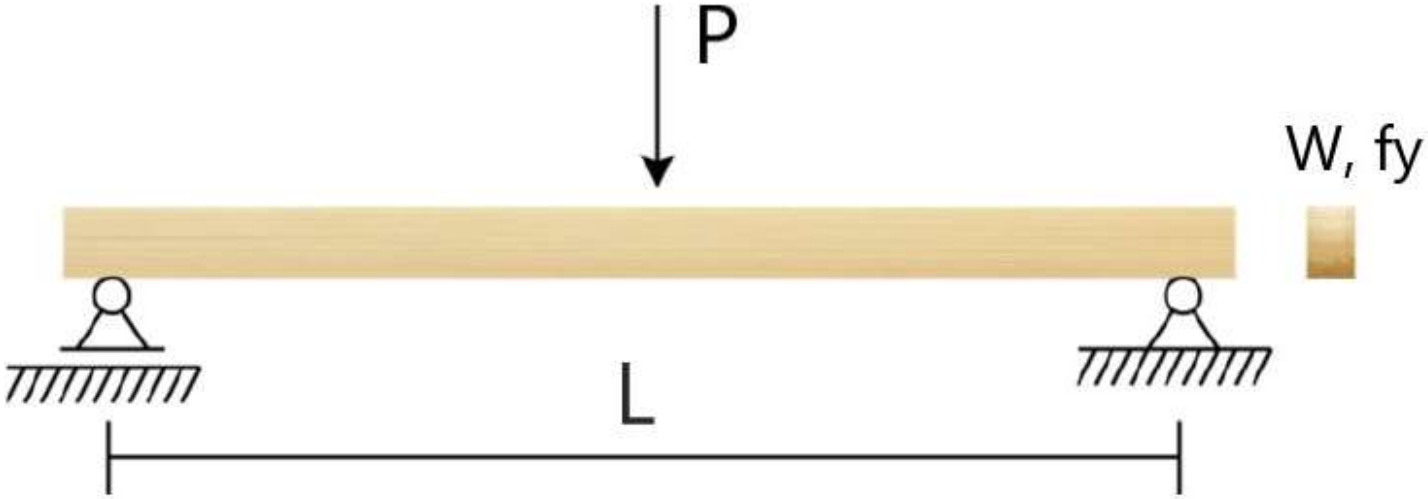
Limit State

$$g(X) = R - S$$

- Limit state functions are not restricted to civil engineering.
- The concept of limit state functions is generic and can be transferred to a large family of problems.

- The parameters considered in the limit state function are subjected to uncertainties.
- How can we include these uncertainties for a «safe» design?

Limit State

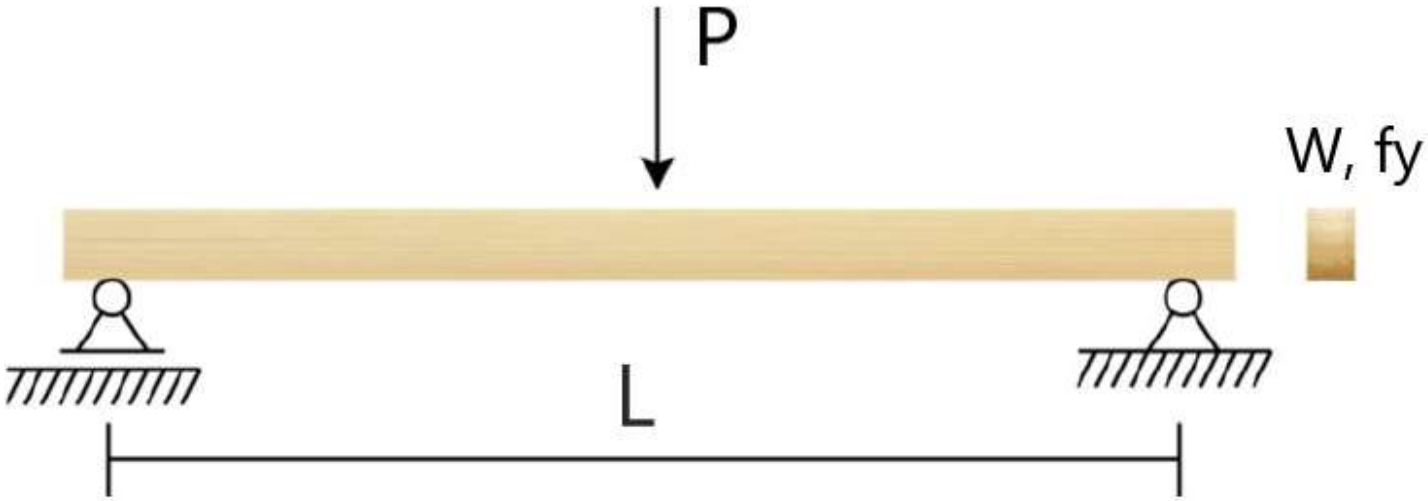


$$R = W \cdot fy \qquad S = \frac{P \cdot L}{4}$$

$$g(X) = R - S = W \cdot fy - \frac{P \cdot L}{4}$$

$$Safe\ domain\ when\ R > S \ \rightarrow g(X) > 0$$
$$Failure\ domain\ when\ R < S \ \rightarrow g(X) < 0$$

Limit State

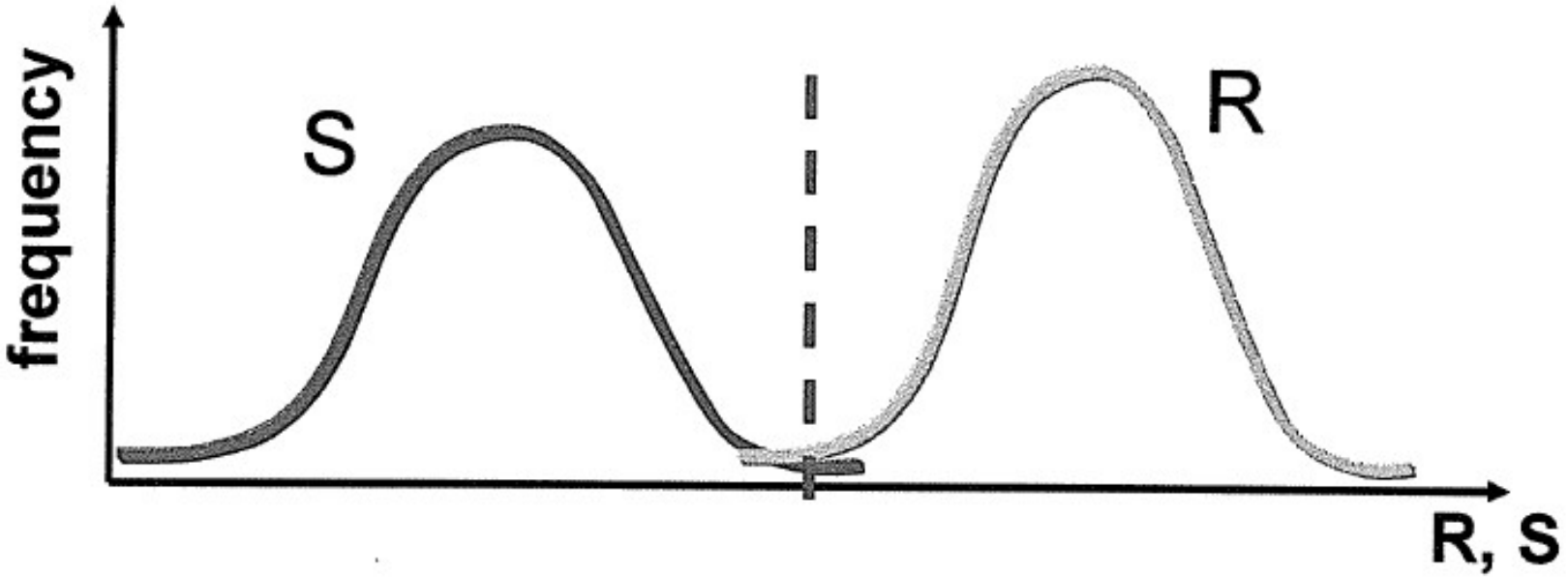


$$Safe\ domain\ when\ R > S \ \rightarrow g(X) > 0$$
$$Failure\ domain\ when\ R < S \ \rightarrow g(X) < 0$$

Probability of failure: $p_f = Pr(R < S)$

Probability of survival $p_s = 1 - p_f$

The Fundamental Case



$$p_f = \text{Prob}\,(R \le S) = \text{Prob}\,(R - S \le 0)$$

If R and S are normally distributed variables $p_f = \Phi(-\beta)$

where $\beta = \dfrac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}}$