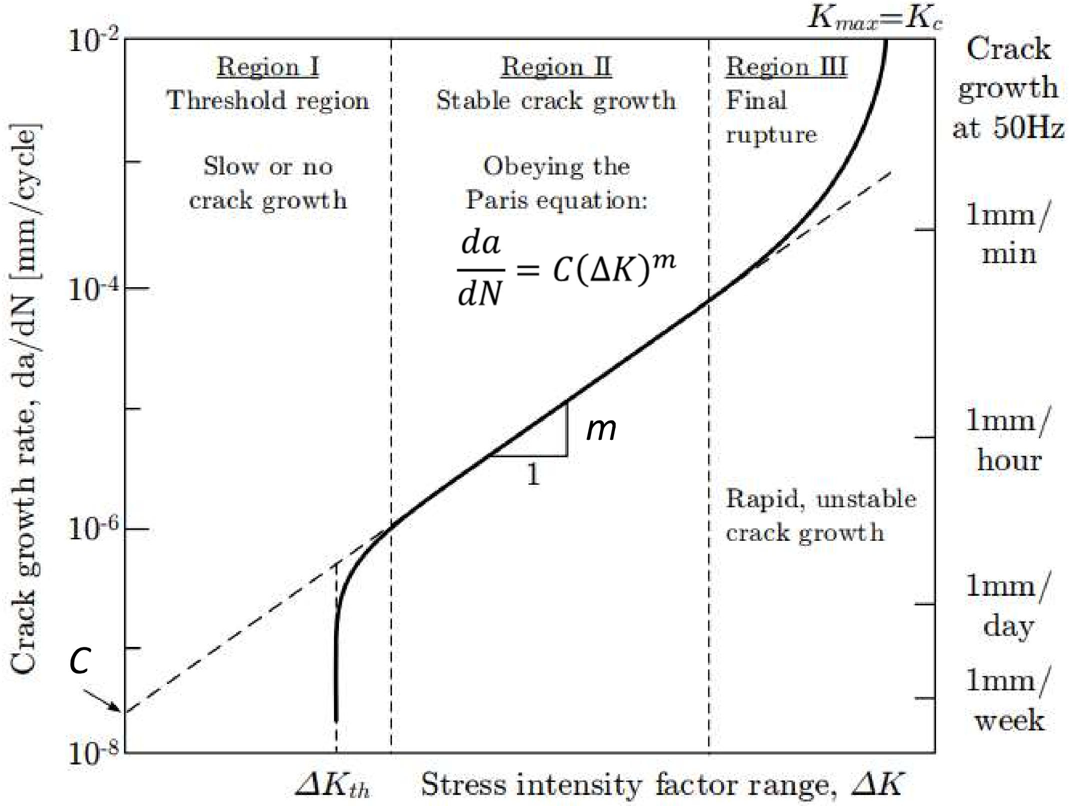
Fatigue analysis of cracks

- Under constant amplitude loading, the rate of crack growth can be estimated as:

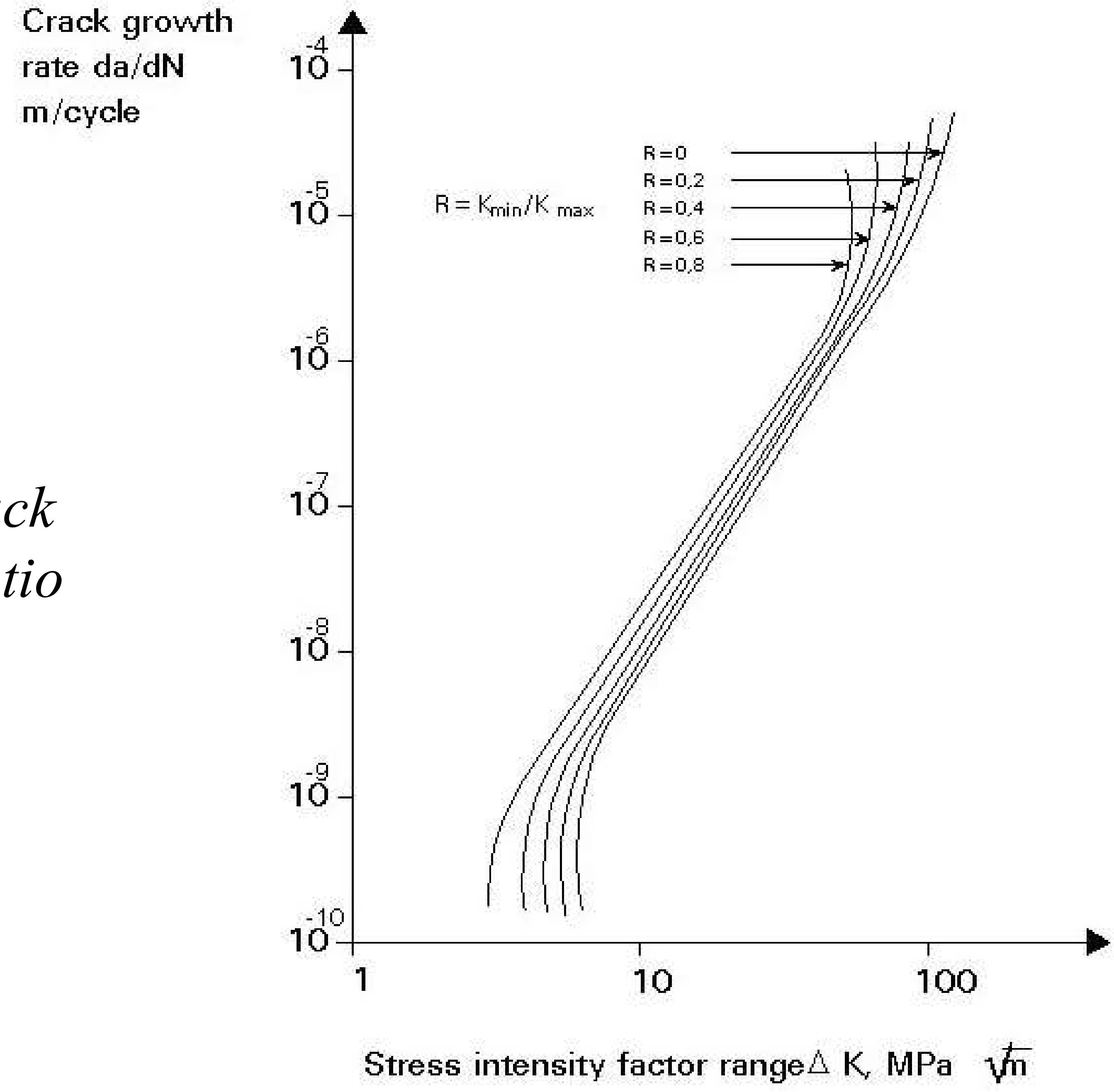
$$\frac{da}{dN} = C(\Delta K)^m \qquad (\textit{Paris-Erdogan law}) \qquad \Delta K = \Delta\sigma \cdot Y \cdot \sqrt{\pi a}$$



- Its position depends on
  i) stress ratio R ($=\sigma_{min} / \sigma_{max}$)
  ii) environment
- Below the threshold value ΔK_{thr} cracks will not propagate
- As fracture is approached, fatigue crack growth accelerates

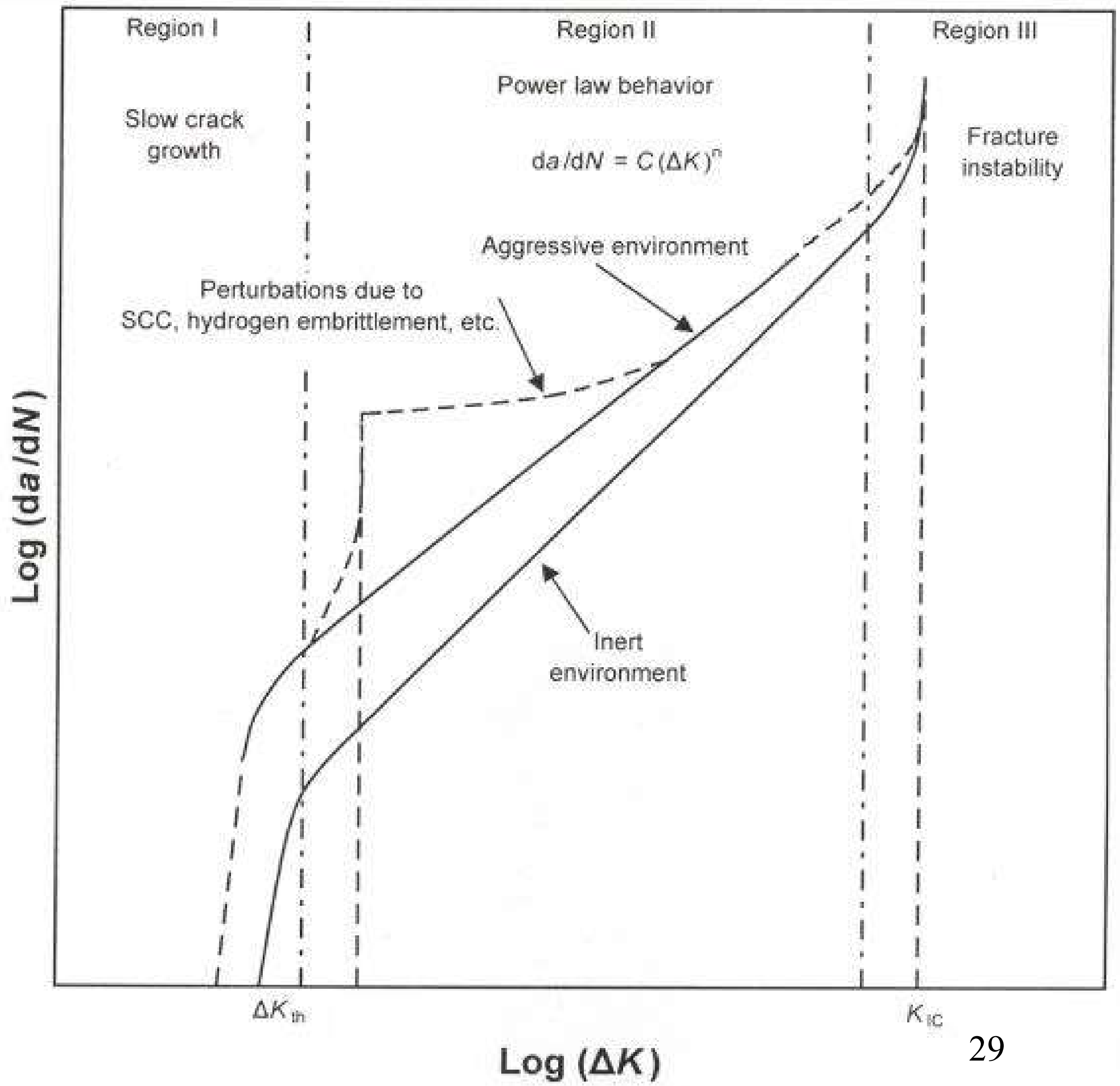
Fatigue analysis of cracks

Dependence of crack growth on stress ratio R (=$\sigma_{min}/\sigma_{max}$)



Fatigue analysis of cracks

Dependence of crack growth on environment



Typical values for Fracture Mechanics parameters

- For welded details, the following values may be used

Material	m	C	ΔK_{th}
Steel	3.0	$5.21 \cdot 10^{-13}$	63
Aluminum	3.0	$1.41 \cdot 10^{-11}$	21

Units: $K : [MPa \cdot \sqrt{mm}]$ and $da/dN : [mm/cycle]$.
Threshold value for high residual stresses: $R = 0.5$.

- For fracture mechanics analysis of riveted details, a similar (conservative) value of ΔK_{th} can be used

$$\Delta K_{th} = 63 N / mm^{3/2} \approx 2 MPa\sqrt{m}$$

(constantly for all R-ratios)

- The material constants C and m for old steel, if not determined in crack propagation tests, can be taken as

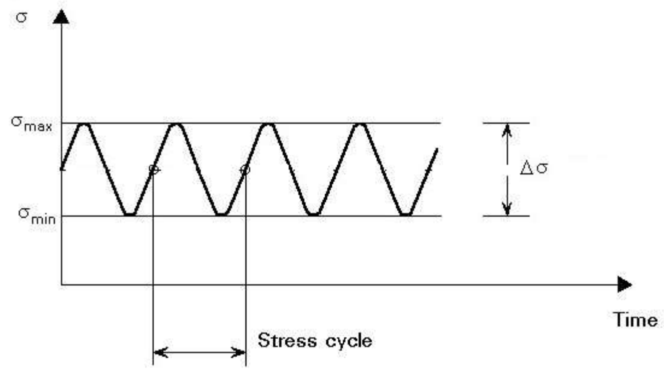
$C = 4 \cdot 10^{-13}$ and $m = 3$ (units in $N(mm)^{-3/2}$ and mm) or
$C = 1.3 \cdot 10^{-11}$ and $m = 3$ (units in $MPa(m)^{1/2}$ and m)

- Wrought iron has higher ΔK_{th} values but the da/dN ratio is also higher in comparison to mild steel.

Fatigue analysis of cracks

$$\frac{da}{dN} = C(\Delta K)^m \qquad \Delta K = \Delta\sigma \cdot Y \cdot \sqrt{\pi a}$$

(*Paris-Erdogan law*)



Solution of the Paris-Erdogan law for constant amplitude loading yields:

$$N_f = \int_{a_i}^{a_f} \frac{da}{C(\Delta K)^m} = \int_{a_i}^{a_f} \frac{da}{C(\Delta\sigma \cdot Y(a) \cdot \sqrt{\pi a})^m} = \int_{a_i}^{a_f} \frac{1}{C(\Delta\sigma \cdot Y \cdot \sqrt{\pi})^m} \frac{da}{a^{m/2}}$$

Since *C*, *Δσ* and *m* are all constant and if *Y* is assumed constant between a_i and a_f, the only variable is *a*, and integration is straightforward, giving the closed-form solution:

$$N_f = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C(\Delta\sigma \cdot Y \cdot \sqrt{\pi})^m (1 - m/2)}$$

In case *Y* cannot be assumed as constant and varies significantly between a_i and a_f, numerical integration becomes necessary to solve the Paris-Erdogan law!

Fatigue analysis of cracks

$$N_f = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C(\Delta\sigma \cdot Y \cdot \sqrt{\pi})^m (1 - m/2)}$$

$$a_f = a_c = \frac{1}{\pi}\left(\frac{K_c}{Y\sigma_{\max}}\right)^2$$

N_f depends on: i) magnitude of the applied stress range ($\Delta\sigma$)
ii) initial crack size (a_i)
iii) final (critical) crack size (a_f)
iv) Y (compare Y for centre-cracked and edge specimens!)

Note that in general:

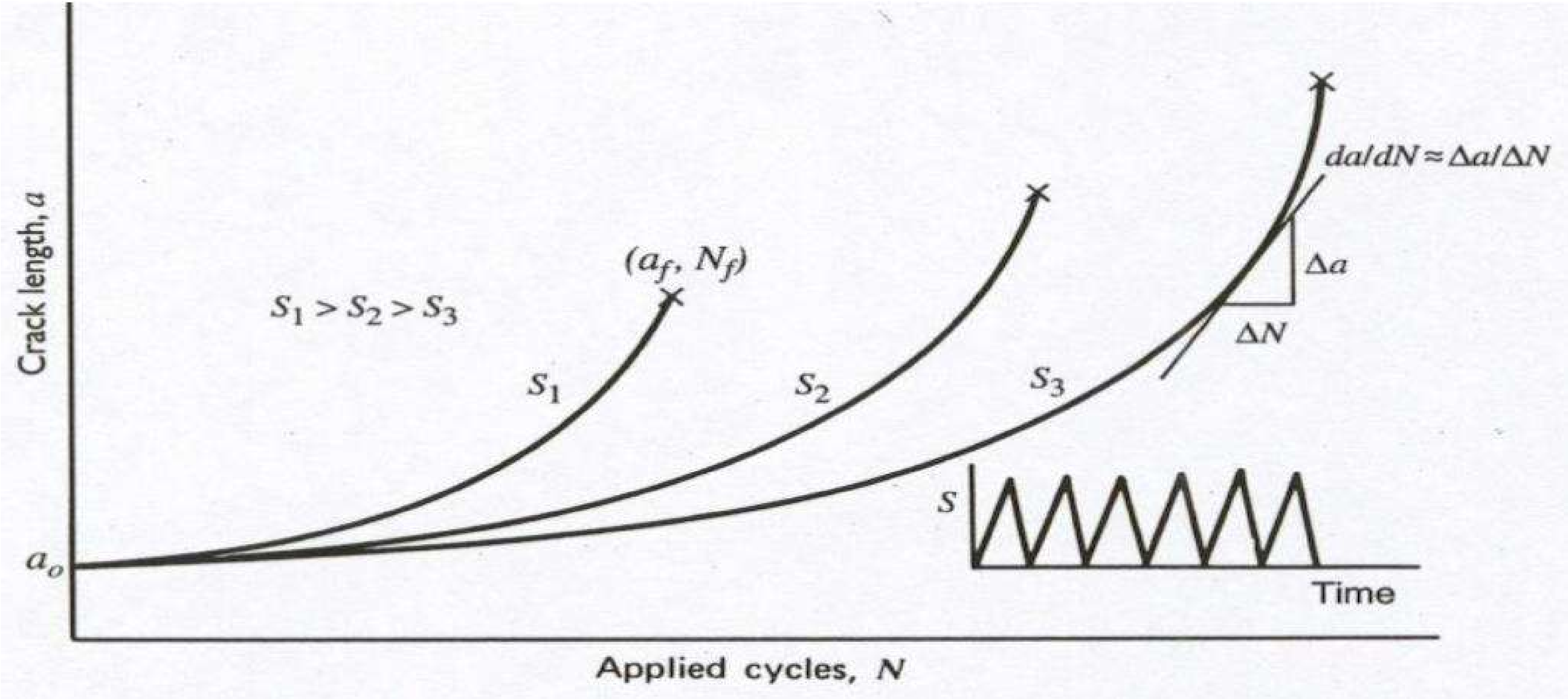
$$N_f = \frac{C}{\Delta\sigma^m}$$ (S-N curve) or $$\log(N_f) = \log(C) - m \cdot log(\Delta\sigma)$$

Fatigue analysis of cracks

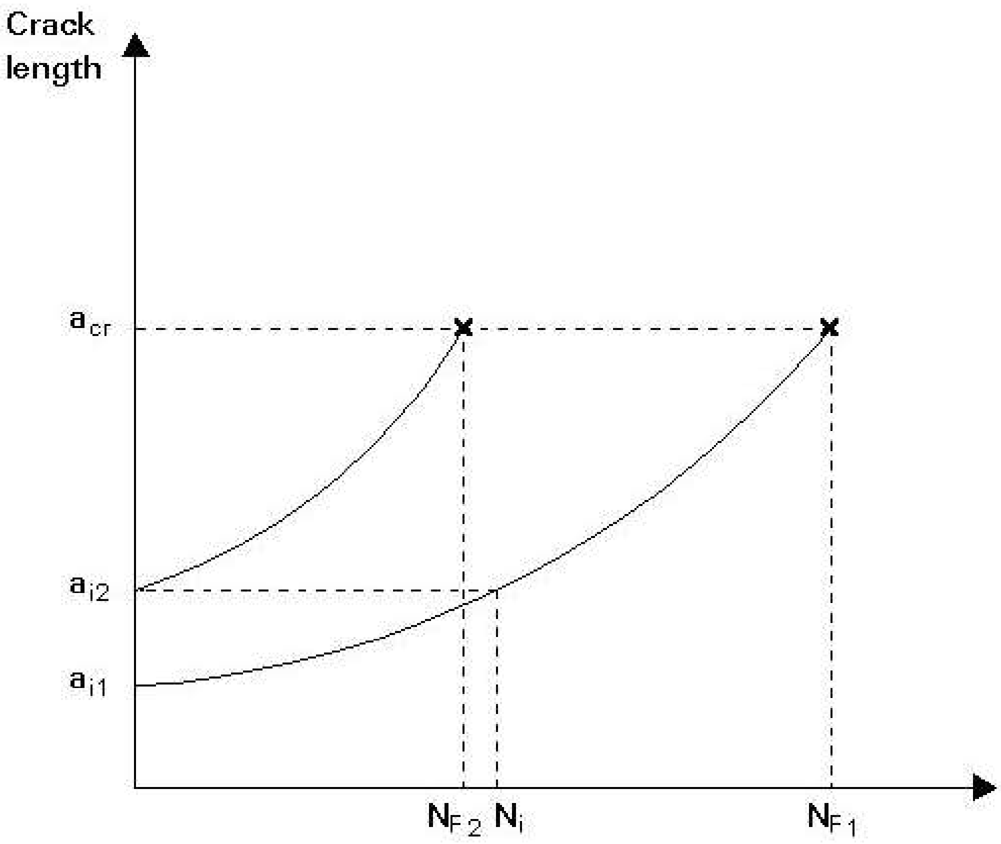
- Several approaches exist for selecting an appropriate value for the initial crack size, a_i:
  - selecting a value based on actual measurements on the structure in question or on a similar structural detail
  - selecting a value based on what is known to be the “minimum detectable crack size” for the method of NDT being employed
  - selecting a value which has been “calibrated” by back-calculation of the fatigue lives of test specimens similar to the details under evaluation

- Several approaches exist for selecting the final or critical crack size a_c or a_f :
  - it can be taken as being equal to the thickness of the cracked element, or some percentage of this thickness.
  - it can be limited by appropriate brittle fracture criteria and calculated from by using the fracture toughness of the material, as shown previously
  - it can be obtained by considering yield criteria using Failure Assessment-Diagrams (FAD)

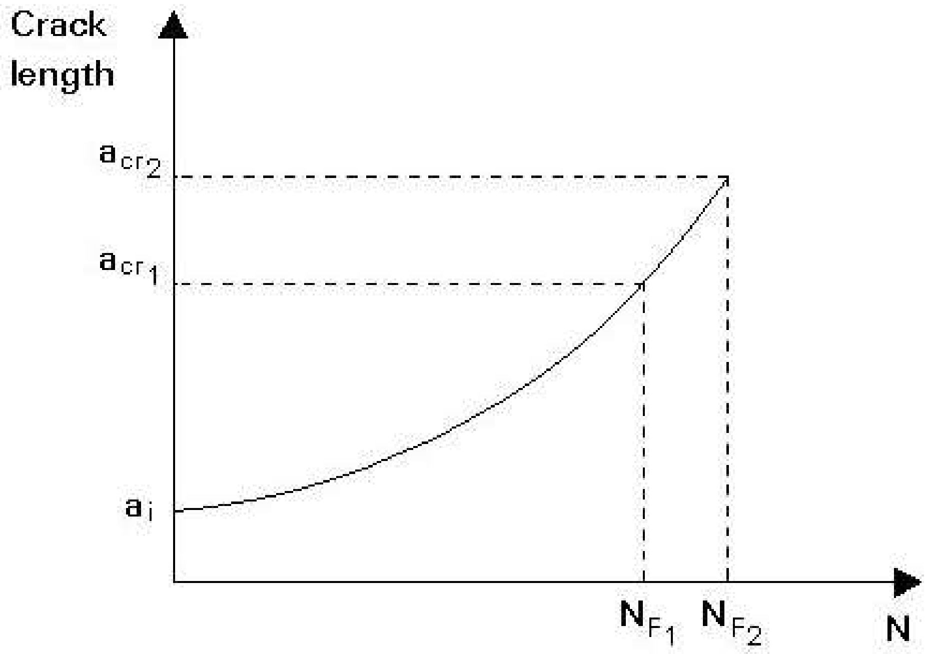
Fatigue analysis of cracks



Fatigue analysis of cracks



Effect of a_i on crack growth



Effect of critical crack size on N_F

Fatigue analysis of cracks

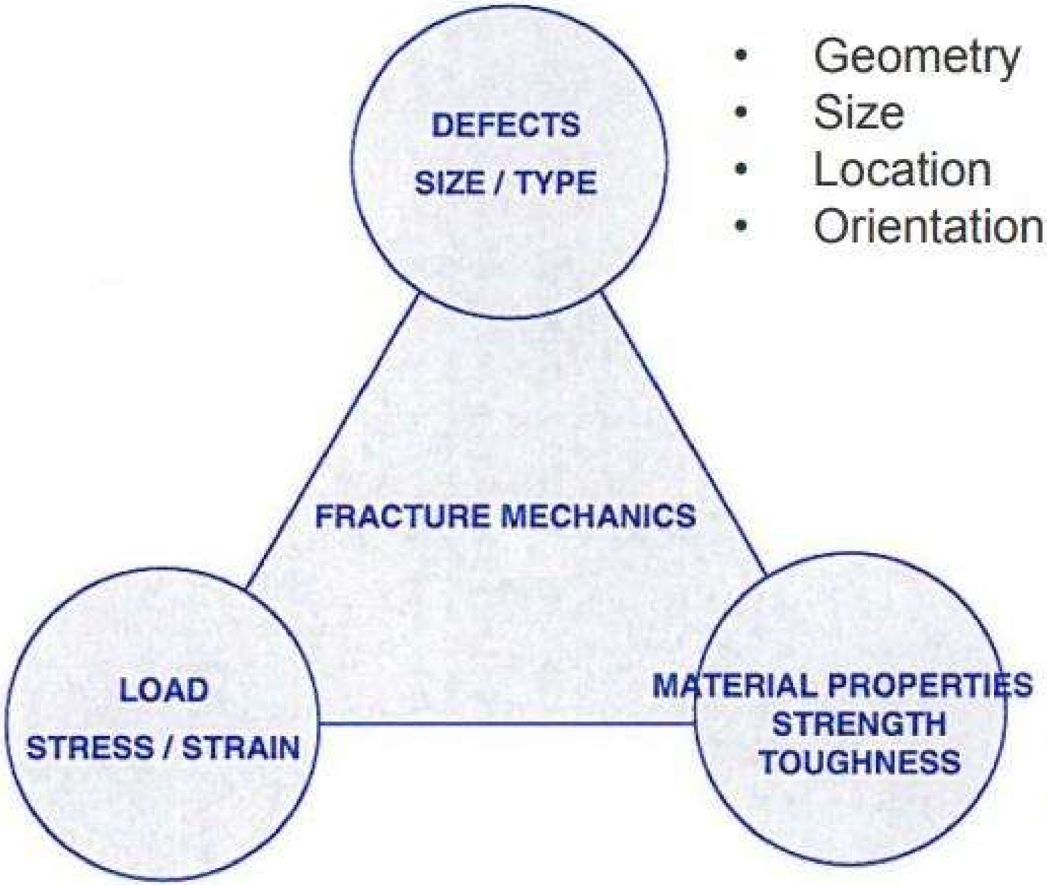
Typical example

An illustrative and simple calculation can be made using the solution of the Paris-Erdogan Law. Consider a small crack in a large structure, which can imply that Y ≈ 1.

Illustrative crack growth life predictions for a carbon steel.

a_0 (mm)	a_f (mm)	crack growth life	ratio
5	50	382	1
5	100	419	1.1
1	50	3781	10

Fracture Mechanics triangle



- Geometry
- Size
- Location
- Orientation

- Primary (Static and Fatigue)
- Secondary (residual stress)

- Fracture toughness
- Fatigue crack growth data

$$K = \sigma \, Y \sqrt{\pi \, a}$$

$$\Delta K = \Delta\sigma \cdot Y \cdot \sqrt{\pi a}$$